Proposal for an Explanation of Limens of Loudness

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It is proposed that the limen of loudness ΔS for pure tones is equal or proportional to the root-mean-square deviation in the number of pulses produced by the sound in the time T of observation. By making a plausible assumption concerning the deviation and by assuming that loudness S is proportional to the pulse rate one obtains $\Delta S = A^{-\frac{1}{2}}T^{-\frac{1}{2}}S^{\frac{1}{2}}$. Here A is a constant which must be a function of frequency if the equation is to fit experimental data. The fit is encouraging but not completely convincing. If valid, the hypothesis might be used to explain other acoustic phenomena.

WHAT determines the size of the limen or jnd of loudness?

If we take successive samples of equal length from a noise generator of "constant" output, these samples will differ in energy. This means that an objective comparison of a noise sample from a particular noise generator of a different intensity may fail to identify the more intense generator correctly. A particular sample from the less intense generator may have more energy than a particular sample from a generator of greater intensity.

This variation of energy from sample to sample must be a factor, if not always a dominating factor, in determining the limen of loudness for noise. Can a somewhat similar factor be responsible for the size of the limen of loudness for pure tones? Here, ideally the objective stimulus is fixed exactly by source intensity. However, the subjective phenomenon of loudness must depend on the nerve pulses which the stimulus produces. A variation in the number of pulses from stimulus to stimulus might be responsible for the size of the limen of loudness.

In the general form presented above, this idea is not new. Van Gemert[1] assumed an additive physiological noise in explaining the size of the limens of loudness for white noise. His work deals with a comparatively narrow range of loudness.

The equation proposed for the limen of loudness ΔS by Stevens[2]

$$\Delta S = k(S+d) \tag{1}$$

can be interpreted as saying that we can judge loudness to a constant fraction k except for the uncertainty introduced by an additive physiological noise or background fluctuation represented by d. If loudness were strictly proportional to intensity, and for $d=0$, Eq. (1) should hold for Gaussian noise if the only uncertainty in judgment arose through the objective uncertainty in the energy of the sample.

In this paper, a somewhat different form of physiological uncertainty or noise is suggested. We note that the signal which is transmitted from the ear to the central nervous system consists of a sequence of nerve pulses. The louder the sound, the greater is the number of pulses per second. While the average number of pulses per second is a function of the sound level, a certain fluctuation in the number of pulses in a fixed time interval might be expected. It is proposed that this fluctuation is responsible for the size of the limen of intensity for sine waves and for its variation with intensity.

We do not know the exact nature of this fluctuation. We can, however, be guided by a simple model. Suppose that pulses are produced randomly and independently at an average rate of N pulses per second. The average number of pulses produced in a time T sec will be NT. The root-mean-square deviation in this number, $(\langle n^2\rangle)^{\frac{1}{2}}$, is given by

$$(\langle n^2\rangle)^{\frac{1}{2}} = (NT)^{\frac{1}{2}}. \tag{2}$$

While we might assume that in the case of nerve pulses the root-mean-square number of pulses will be some constant factor times $(NT)^{\frac{1}{2}}$, we will for the sake of simplicity assume that Eq. (2) holds for the nerve pulses produced by the ear.

We will further assume that in some way the nervous system bases its estimate of loudness on the number of pulses produced in same time T, and that the liminal number of pulses during same time T, which we may express as $T\Delta N$, is equal to $(\langle n^2\rangle)^{\frac{1}{2}}$. Thus,

$$\begin{aligned} T\Delta N &= (NT)^{\frac{1}{2}} \\ \Delta N &= T^{-\frac{1}{2}}N^{\frac{1}{2}}. \end{aligned} \tag{3}$$

Here T is a time of observation or integration, characteristic of the organism and the experiment.

Again, we might insert a constant multiplying factor in this Eq. (3), but for the sake of simplicity we refrain from doing so.

In order to test our hypothesis, we must relate the number of pulses per second to some measurable quantity. Fletcher proposes that loudness in sones is proportional to the average number of pulses per second.[3] Let us adopt this suggestion and say that the number of pulses per second N is a constant A times

[1] A. G. M. Van Gemert, Acta Physiol. Pharmacol. Neerl. 4, 274–290 (1955).
[2] S. S. Stevens, Psychol. Rev. 64, 153–181 (1957).
[3] Harvey Fletcher, *Speech and Hearing in Communication* (D. Van Nostrand Company, Inc., Princeton, 1953), Chap. 14.

the loudness in S in sones;

$$N=AS. \tag{4}$$

From Eqs. (4) and (3) we have for the loudness limen ΔS in sones

$$\Delta S=A^{-\frac{1}{2}}T^{-\frac{1}{2}}S^{\frac{1}{2}}. \tag{5}$$

Limens are customarily given as a change ΔI in intensity at a given intensity I called sensation level, which is measured in decibels above the threshold of hearing. By means of the Fletcher-Munson[4] curves we can convert sensation level I to loudness level P measured in phons. Then by means of Fletcher's Fig. 137,[4] we can convert loudness level in phons to loudness in sones. If ΔI is the liminal difference in intensity or sensation level at a given sensation level and frequency, then the liminal loudness difference ΔS in sones will be

$$\Delta S=\Delta I\left(\frac{\partial P}{\partial I}\right)\left(\frac{\partial \ln S}{\partial P}\right)S. \tag{6}$$

The quantity $\partial \ln S/\partial P$ as a function of P was obtained graphically from Fletcher's Fig. 137. The quantity $\partial P/\partial I$ was obtained graphically from plots of P *vs* I for various frequencies; the plots were made from Fletcher's Fig. 134.[4] The liminal differences in intensity of Riesz[5] were used. Thus, ΔS could be plotted *vs* S for various frequencies.

Let us first consider the case of a frequency of 1000 cps. For this frequency, by definition $P=I$, and $\partial P/\partial I=1$. Thus, at 1000 cy there can be no error associated with P as a function of I.

The points in Fig. 1 represent Riesz's experimental values. The solid line follows Eq. (5). While (5) does not agree with experiment above a loudness of 10 sones, the agreement is good over more than 3 decades of S below this loudness. The data for loudnesses above 10 sones agree reasonably with a law that ΔS is proportional to S, as in Eq. (1) with $d=0$, as shown by the dashed line.

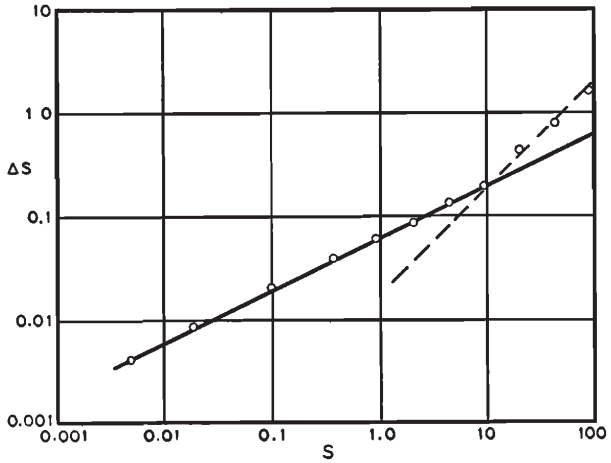

FIG. 1. Limens of loudness *vs* loudness at 1000 cy.

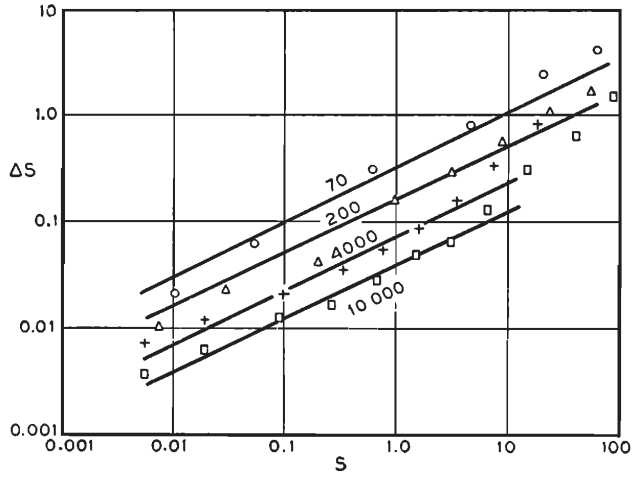

FIG. 2. Limens of loudness *vs* loudness with frequency as a parameter.

Let us now consider behavior at other frequencies. Figure 2 shows an attempt to fit lines representing Eq. (5) to Riesz's data for frequencies of 70, 200, 4000, and 10000 cps. The points are more scattered and the fit is not nearly as good as for 1000 cy. It is possible that this might be due either to errors in measurement or to errors in deriving $\partial P/\partial I$ from the Fletcher-Munson curves.

There is another reason why the experimental data might not agree with Eq. (5) even if (5) were valid. Unless all subjects were identical, (5) could be expected to hold for individual subjects only. However, Riesz's data, the Fletcher-Munson curves, and the curve relating sones and phons are each averages over a group of subjects. Even if (5) held for an individual subject it would not necessarily hold for average data representing groups of subjects.

Stevens[6] gives data for log of the number of difference limens *vs* log of loudness in sones. We can obtain a theoretical value for this relation from Eq. (5). In the loudness range dS the number of limens dM is given by

$$dM=\frac{dS}{\Delta S}=A^{\frac{1}{2}}T^{\frac{1}{2}}S^{-\frac{1}{2}}dS. \tag{7}$$

By integrating and letting $M=0$ at $S=0$, we obtain

$$M=2A^{\frac{1}{2}}T^{\frac{1}{2}}S^{\frac{1}{2}}. \tag{8}$$

In Fig. 3, lines of slope $\frac{1}{2}$, in accord with Eq. (8), are fitted to the points given by Stevens. The fit is good at lower loudnesses but less good at greater loudnesses.

Stevens' points were obtained using Riesz's limens, the Fletcher-Munson curves, and a subjective loudness curve of Churcher[7] which does not differ much from Fletcher's scale. The fact that the fit is somewhat

[4] Reference 3, Chap. 11.
[5] Reference 3, Chap. 9.
[6] S. S. Stevens and H. Davis, *Hearing* (John Wiley and Sons, Inc., New York, 1938), Fig. 61, p. 150.
[7] B. G. Churcher, J. Acoust. Soc. Am. **6**, 216–226 (1935).

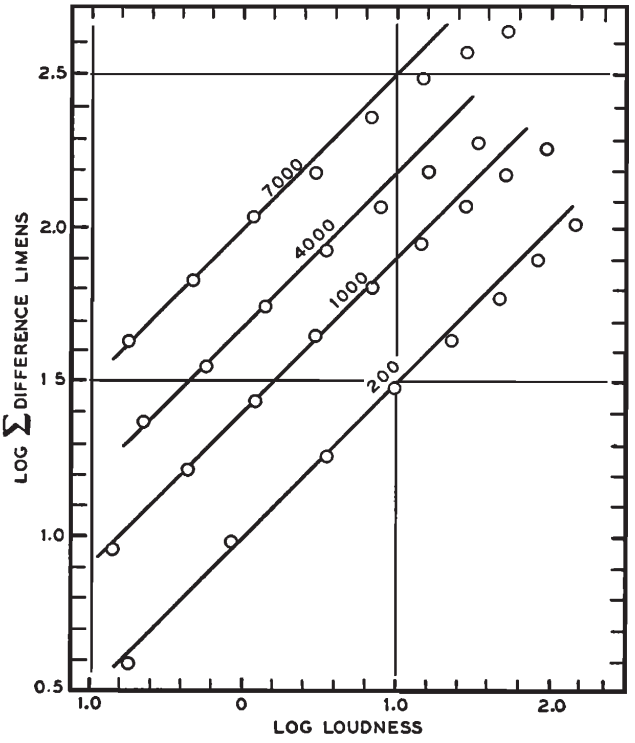

FIG. 3. Number of limens *vs* loudness for various frequencies.

better in Fig. 3 than in Fig. 2 may reflect a more successful graphical differentiation of the Fletcher-Munson curves.

Stevens fitted the points of Fig. 3 with the empirical curve

$$S=KM^{2.2}$$

$$M=K^{-0.455}S^{0.455}. \tag{9}$$

Here K is a constant. This empirical relation does not fit much better than Eq. (8). The fit of Eq. (9) is less good in the range from 0.1 to perhaps 1 sone, but better at 10 sones. In Fig. 1 we have seen a departure from Eq. (5) and hence from Eq. (8) above 10 sones.

We may inquire at this point why Fletcher's loudness scale was used in preference to that proposed, for instance, by Stevens.[8] In order to make a meaningful test of Eqs. (5) and (8) we must use an experimental curve which is well enough defined so that its derivative can be plausibly taken. It is clear from the data shown by Stevens[8] that the methods to which he has limited himself do not define a curve with this degree of certainty. While arguments may be advanced concerning the psychological meaning of the bases of Fletcher's subjective loudness curve, at least the experiments performed do define a curve within close limits.

[8] S. S. Stevens, J. Acoust. Soc. Am. **27**, 815–829 (1955).

We may also note concerning Riesz's limens, that while they precede the modern definition of the limen as the change which is correctly identified 75% of the time, and hence may differ from other limens by a constant factor, there seems no reason to doubt their reliability or consistency. Like Fletcher's subjective loudness data, they refer to the organism's response to the total excitation of the cochlea, and hence perhaps to the total production of nerve pulses by a tone of a particular frequency, and thus seem appropriate in connection with Eq. (6).

Let us now return to Eq. (5) and the hypotheses which led to it. One of these is that loudness in sones is proportional to number of pulses per second. Figure 2 convinces us that if this is so the constant of proportionality must be different for different frequencies. It could be that there is a division or multiplication of the pulse rate by different factors in the nerve pathway between the ear and whatever point the loudness is judged. Such a change could maintain the ratio of the root-mean-square deviation in number of pulses in a time T to the number of pulses in the time T, while changing the number of pulses. This would have the effect of introducing a factor dependent on frequency only in Eq. (5).

If we take Eq. (5) and Fig. 1 literally, we can evaluate the factor $T^{-\frac{1}{2}}A^{-\frac{1}{2}}$. We find for a frequency of 1000 cy

$$T^{-\frac{1}{2}}A^{-\frac{1}{2}}=0.061.$$

The intensity limen data used were obtained by a beat method, with about 3 beats a sec. We might argue that the soft sound is on for about $\frac{1}{6}$ sec followed by the loud sound for about $\frac{1}{6}$ sec, and take T to be $\frac{1}{6}$ sec. This leads to a value for A

$$A=1610.$$

According to this, a 1000-cy tone 100 db above threshold would correspond to 161 000 pulses per sec, while 5 db above threshold the pulse rate would be 8 pulses per sec. The organ of Corti is said to be innervated by some 30 000 neurons. The pulse rates do not seem ridiculous, but we have little reason to believe that they are realistic.

In order to examine the general validity of the hypothesis presented here, it would be necessary to compare Eq. (5) with very accurate data on single subjects, and preferably data in which a clear meaning could be given to the observation or integration time T. It is perhaps idle to make more elaborate analyses based on the hypothesis or to speculate on its broader application until we know how accurately the conclusions fit experimental results.